

Δ -string - a hybrid between wormhole and string

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Abstract

The flux tube solutions in 5D Kaluza-Klein theory can be considered as a string-like object - Δ -string. The initial 5D metric can be reduced to some inner degrees of freedom living on the Δ -string. The propagation of electromagnetic waves through the Δ -string is considered. It is shown that the difference between Δ and ordinary strings are connected with the fact that for the Δ -string such limitations as critical dimensions are missing.

1 Introduction

The difference between point-like particles and strings on the one hand, and Einstein's point of view on an inner structure of matter on the other hand is that according to Einstein everything must have an inner structure. Even more : at the origin of matter should be vacuum. In string theory a string is a vibrating 1-dimensional object and the string has many different harmonics of vibration, and in this context different elementary particles are interpreted as different harmonics of the string. The string degrees of freedom are the coordinates of string points in an ambient space.

In this paper we would like to consider the situation when the string has an inner structure. The question in this situation is : what will be changed in this situation ? Definitely we can say that in this situation the string is an object which effectively arise from a field theory and such object has inner degrees of freedom which are not connected with an external space. As a model of such kind of string-like object we will consider gravitational flux tubes. These tubes are the solutions in 5D Kaluza-Klein gravity [1] filled with electric E and magnetic H fields. If $E = H$ we have an infinite tube, if $E \approx H$ (but $E > H$) the length of the tube can be arbitrary long and the cross section can be $\approx l_{Pl}$ (l_{Pl} is the Planck length) [2]. Such flux tube can be considered as a string attached to two Universes or to remote parts of a single Universe. For the observer in the outer Universe the attachment points looks like to point-like electric and magnetic charges (see Fig. 1).

We have to note that similar construction was presented in Ref. [3] : the matching of two remote regions was done using 4D infinite flux tube which is the Levi-Civita - Bertotti - Robinson solution [4], [5] filled with the electric and magnetic fields.

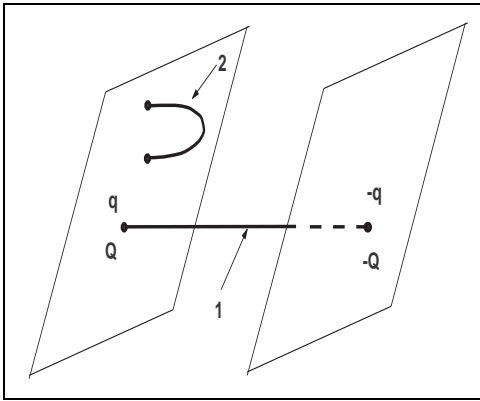


Figure 1: The gravitational flux tube attached to two Universes (1) or to remote parts of a single Universe (2). The cross section is in order of the Planck length and the tube length can be arbitrary long. Such construction is similar to the string attached to D-brane(s).

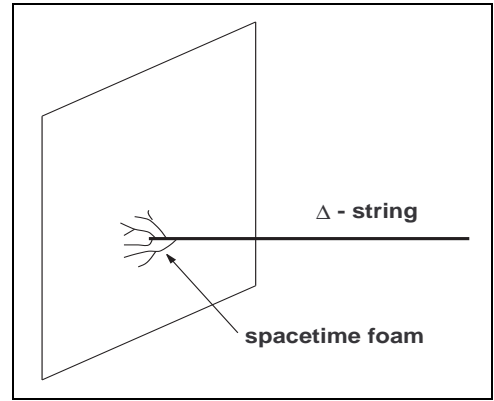


Figure 2: Δ -string is the gravitational flux tube surrounded with the handles of spacetime foam at the ends.

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2 Gravitational flux tube

In this section we will describe the gravitational flux tube and why it can be considered as a Δ -string. Let us consider the following 5D metric

$$ds^2 = e^{2\nu(r)} dt^2 - e^{2\psi(r)-2\nu(r)} [d\chi + \omega(r)dt + Q \cos \theta d\varphi]^2 - dr^2 - a(r)(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

where χ is the 5th extra coordinate; r, θ, φ are 3D spherical-polar coordinates; Q is the magnetic charge. The solution of 5D Kaluza-Klein equations depends on the relation $\delta = 1 - H/E$ between electric E and magnetic H fields. If $\delta = 0$ we have an infinite flux tube filled with electric and magnetic fields [1]

$$a_0 = \frac{q^2}{2} = \text{const}, \quad (2)$$

$$e^\psi = e^\nu = \cosh \frac{r\sqrt{2}}{q}, \quad (3)$$

$$\omega = \sqrt{2} \sinh \frac{r\sqrt{2}}{q} \quad (4)$$

here q is the electric charge. If $0 < \delta \ll 1$ the part of spacetime located between two hypersurfaces $ds^2(r = \pm r_H) = 0$ ¹ is a finite flux tube filled with electric and magnetic fields. In both cases the cross sectional sizes can be arbitrary but we choose its in the Planck region ($\approx l_{Pl}^2$). This condition is very important for the idea presented here : the gravitational flux tube can be considered as the Δ -string attached to two different Universes (or to remote parts of a single Universe). From the physical point of view this is 1D object as the Planck length is a minimal length in the physical world.

Now our strategy is to consider small perturbations of 5D metric. Generally speaking they are 5D gravitational waves on the gravitational flux tube (or on the string language - vibrations of Δ -string). In the general case 5D metric is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - \phi^2 (d\chi + A_\mu dx^\mu)^2 \quad (5)$$

here $g_{\mu\nu}$ is the 4D metric; $\mu, \nu = 0, 1, 2, 3$; A_μ is the electromagnetic potential; ϕ is the scalar field. The corresponding 5D Kaluza-Klein's equations are (for the reference see, for example, [7])

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{\phi^2}{2} T_{\mu\nu} - \frac{1}{\phi} [\nabla_\mu (\partial_\nu \phi) - g_{\mu\nu} \square \phi], \quad (6)$$

$$\nabla_\nu F^{\mu\nu} = -3 \frac{\partial_\nu \phi}{\phi} F^{\mu\nu}, \quad (7)$$

$$\square \phi = -\frac{\phi^3}{4} F^{\alpha\beta} F_{\alpha\beta} \quad (8)$$

where $R_{\mu\nu}$ is the 4D Ricci tensor; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the 4D Maxwell tensor and $T_{\mu\nu}$ is the energy-momentum tensor for the electromagnetic field.

We will consider only δA_μ perturbations, $\delta g_{\mu\nu}$ and $\delta \phi$ degrees of freedom are frozen. For this approximation we have equation

$$\nabla_\nu \delta F^{\mu\nu} = 0. \quad (9)$$

We introduce only one small perturbation in the electromagnetic potential

$$\delta A_\theta = f(t, r) \quad (10)$$

since for the background metric $\phi = \text{const}$. Generally speaking A_θ -component should have some dependence on the θ -angle. But the cross section of the Δ -string is in Planck region and consequently the points with different θ and φ ($r = \text{const}$) physically are not distinguishable. Therefore all physical quantities on the Δ -string should be averaged over polar angles θ and φ . It means that the δA_θ in Eq. (10) is averaged quantity.

After this (very essential) remark we have the following wave equation for the function $f(t, r)$

$$\partial_{tt} f(t, x) - \cosh x \partial_x \left(\cosh x \partial_x f(t, x) \right) = 0 \quad (11)$$

here we have introduced the dimensionless variables $t/\sqrt{a_0} \rightarrow t$ and $r/\sqrt{a_0} \rightarrow x$. The solution is

$$f(t, x) = f_0 F(t - 2 \arctan e^x) + f_1 F(t + 2 \arctan e^x) \quad (12)$$

here $f_{0,1}$ are some constants and F and H are arbitrary functions. This solution has more suitable form if we introduce new coordinate $y = 2 \arctan e^x$. Then

$$f(t, y) = f_0 F(t - y) + f_1 H(t + y). \quad (13)$$

¹at these points $\exp(-2\nu(r = \pm r_H)) = 0$

The metric is

$$ds^2 = \frac{a_0}{\sin^2 y} dt^2 - \frac{dy^2}{\sin^2 y} - a_0 (d\theta^2 + \sin^2 \theta d\varphi^2) - (d\chi + \omega dt + Q \cos \theta d\varphi)^2. \quad (14)$$

Thus the simplest solution is electromagnetic waves moving in both directions along the Δ -string.

3 The comparison with string theory

Now we want to compare this situation with the situation in string theory. How is the difference between the result presented here (Eq. (11), (12)) and the string oscillation in the ordinary string theory? The action for bosonic string is

$$S = -\frac{T}{2} \int d^2 \sigma \sqrt{-h} h^{ab} \partial_a \mathcal{X}^\mu \partial_b \mathcal{X}_\mu \quad (15)$$

here $\sigma^a = \{\sigma, \tau\}$ is the coordinates on the world sheet of string; \mathcal{X}^μ are the string coordinates in the ambient spacetime; h_{ab} is the metric on the world sheet. The variation with respect to \mathcal{X}^μ give us the usual 2D wave equation

$$\square \mathcal{X}^\mu \equiv \left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) \mathcal{X}^\mu = 0 \quad (16)$$

which is similar to Eq. (11). The difference is that the variation of the action (15) with respect to the metric h^{ab} gives us some constraints equations in string theory

$$T_{ab} = \partial_a \mathcal{X}^\mu \partial_b \mathcal{X}_\mu - \frac{1}{2} h_{ab} h^{cd} \partial_c \mathcal{X}^\mu \partial_d \mathcal{X}_\mu = 0 \quad (17)$$

but for Δ -string analogous variation gives the dynamical equation for 2D metric h_{ab} .

The more detailed description is the following. The topology of 5D Kaluza-Klein spacetime is $M^2 \times S^2 \times S^1$ where M^2 is the 2D space-time spanned on the time and longitudinal coordinate r ; S^2 is the cross section of the flux tube solution and it is spanned on the ordinary spherical coordinates θ and φ ; $S^1 = U(1)$ is the Abelian gauge group which in this consideration is the 5th dimension. The initial 5D action for Δ -string is

$$S = \int \sqrt{-\overset{(5)}{g}} \overset{(5)}{R} \quad (18)$$

where $\overset{(5)}{g}$ is the 5D metric (5); $\overset{(5)}{R}$ are 5D Ricci scalars. We want to reduce the initial 5D Lagrangian (18) to a 2D Lagrangian (here we will follow to Ref. [6]). Our basic assumption is that the sizes of 5th and dimensions spanned on polar angles θ and φ is approximately $\approx l_{Pl}$. At first we have usual 5D \rightarrow 4D Kaluza-Klein dimensional reduction. Following, for example, to review [7] we have

$$\frac{1}{16\pi} \frac{\overset{(5)}{R}}{\overset{(5)}{G}} = \frac{1}{16\pi G} \overset{(4)}{R} - \frac{1}{4} \phi^2 F_{\mu\nu} F^{\mu\nu} + \frac{2}{3} \frac{\partial_\alpha \phi \partial^\alpha \phi}{\phi^2} \quad (19)$$

where $\overset{(5)}{G} = G \int dx^5$ is 5D gravitational constant; G is 4D gravitational constant; $\overset{(4)}{R}$ is the 4D Ricci scalars. The determinants of 5D and 4D metrics are connected as $\overset{(5)}{g} = \overset{(4)}{g} \phi$. One of the basis paradigm of quantum gravity is that a minimal length in the Nature is the Planck scale. Physically it means that not any physical fields depend on the 5th, θ and φ coordinates.

The next step is reduction from 4D to 2D. The 4D metric can be expressed as

$$\begin{aligned} d s^2 &= g_{\mu\nu} dx^\mu dx^\nu = g_{ab}(x^c) dx^a dx^b + \\ &\chi(x^c) \left(\omega_{\bar{i}}^{\bar{j}} dy^{\bar{i}} + B_{\bar{a}}^{\bar{i}}(x^c) dx^{\bar{a}} \right) \left(\omega_{\bar{j}}^{\bar{j}} dy^{\bar{j}} + B_{\bar{i}\bar{a}}(x^c) dx^{\bar{a}} \right) \eta_{\bar{i}\bar{j}} \end{aligned} \quad (20)$$

where $a, b, c = 0, 1$; x^a are the time and longitudinal coordinates; $-\left(\omega_{\bar{i}}^{\bar{j}} dy^{\bar{i}} \right) \left(\omega_{\bar{j}}^{\bar{j}} dy^{\bar{j}} \right) \eta_{\bar{i}\bar{j}} = dl^2$ is the metric on the 2D sphere S^2 ; $y^{\bar{i}}$ are the coordinates on the 2D sphere S^2 ; all physical quantities g_{ab} , χ and $B_{\bar{i}\bar{a}}$ can depend only on the physical coordinates x^a . Accordingly to Ref. [8] we have the following dimensional reduction to 2 dimensions

$$\begin{aligned} \overset{(4)}{R} &= \overset{(2)}{R} + R(S^2) - \frac{1}{4} \phi_{\bar{a}\bar{b}}^{\bar{i}} \phi_{\bar{i}}^{\bar{a}\bar{b}} - \\ &\frac{1}{2} h^{ij} h^{kl} (D_a h_{ik} D^a h_{jl} + D_a h_{ij} D^a h^{kl}) - \nabla^a (h^{ij} D_a h_{ij}) \end{aligned} \quad (21)$$

where $R^{(2)}$ is the Ricci scalar of 2D spacetime; D_μ and $\phi_{ab}^{\bar{i}}$ are, respectively, the covariant derivative and the curvature of the principal connection $B_a^{\bar{i}}$ and $R(S^2)$ is the Ricci scalar of the sphere $S^2 = \text{SU}(2)/\text{U}(1)$ with linear sizes $\approx l_{Pl}$; h_{ij} is the metric on \mathcal{L}

$$\mathfrak{su}(2) = \text{Lie}(\text{SU}(2)) = \mathfrak{u}(1) \oplus \mathcal{L}, \quad (22)$$

$$\mathfrak{u}(1) = \text{Lie}(\text{U}(1)) \quad (23)$$

here \mathcal{L} is the orthogonal complement of the $\mathfrak{u}(1)$ algebra in the $\mathfrak{su}(2)$ algebra; the index $i \in \mathcal{L}$. The metric h_{ij} is proportional to the scalar χ in Eq. (17).

The 2D action (which is our goal) is

$$S = \int d^2x^a \phi (\det g_{ab}) \left(\det \omega_i^{\bar{i}} \right) \left[\frac{R^{(2)}}{16\pi G} - \frac{1}{4} \phi^2 (F_{t\theta} F^{t\theta} + F_{r\theta} F^{r\theta}) + \text{other terms} \right]. \quad (24)$$

The second term in the [...] brackets give us the wave equation (11) and the most important is that the first term is not total derivative in contrast with the situation in ordinary string theory in the consequence of the factors ϕ and $(\det \omega_i^{\bar{i}})$. Therefore the variation with respect to 2D metric g_{ab} give us some dynamical equations contrary to string theory where this variation leads to the constraint equations (17).

This remark allows us to say that the Δ -string do not have such peculiarities as critical dimensions (D=26 for bosonic string). The reason for this is very simple : the comprehending space for Δ -string is so small that it coincides with Δ -string. Thus we can suppose that the critical dimensions in string theory is connected with the fact that the string curves the external space but the back reaction of curved space on the metric of the string world sheet is not taken into account.

4 Discussion and conclusions

Finally : the most important difference between Δ and ordinary strings is that in the first case the dynamical equation for 2D metric is replaced by the constraint equation for the second case. Physically it means that in the second case we ignore the back reaction of string on the metric of world sheet. It is supposed that in ordinary string theory this back reaction is zero but for the Δ -string this reaction is very big since the string coincides with the ambient space.

One can say that the Δ -string is a hybrid between the wormhole and the string as it is the wormhole-like solution in 5D Kaluza-Klein gravity on the one hand and it is approximately 1D object on the other hand.

For the outer observer the attachment point of Δ -string to outer Universe looks like a distributed electric and magnetic charges as this point is spread in the consequence of the appearance of spacetime foam handles between Δ -string and the outer Universe. The Δ -string is a bridge between two Universes (or remote parts of a single Universe) like to wormhole on the one hand and has an arbitrary long throat like to the string. The Δ -string can be considered as a model of Wheeler's "mass without mass" and "charge without charge". We have shown that such object can transfer the electromagnetic waves from one Universe to another one or from one part of Universe to another one.

The discussion of electromagnetic waves propagating through the Δ -string is not full as we have frozen $\delta g_{\mu\nu}$ and $\delta\phi$ perturbations. Evidently δA_μ perturbations will initiate $\delta g_{\mu\nu}$ and $\delta\phi$ waves and certainly the back reaction takes place.

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